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Chemical Engineering Journal 84 (2001) 565–571

Chemical Engineering Journal

www.elsevier.com/locate/cej

Short communication

Numerical investigation on dispersive mixing characteristics of MAXBLEND and double helical ribbons

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Received 20 March 2000; received in revised form 22 January 2001; accepted 22 January 2001

Abstract

The local and total dispersive mixing performance of large type impellers, a standard type of MAXBLEND and double helical ribbons impellers, were numerically analyzed using two indices, local dispersive mixing efficiency and NPD function. The results indicated that a standard type of MAXBLEND has a satisfactory local dispersive mixing performance, especially in the grid region where the local dispersive mixing efficiency is high to near 1. However, when the *Re* is somewhat low, the total dispersive mixing performance is not as satisfactory as that operated under a moderate *Re* number. The double helical ribbon impeller can not provide a promising local mixing performance, although it can induce a good total circulation throughout the stirring tank.

Keywords: Numerical simulation; Local dispersive mixing efficiency; NPD function; MAXBLEND impeller; DHR impeller

1. Introduction

Viscous dispersive mixing is an important unit operation in polymerization, food and other industrial processes. To respond to the needs of industrial processes, various impellers have been developed in recent years. MAXBLEND (see Fig. 1) is one of the popular large shape impellers in Japan because of its rather good distributive and dispersive mixing performance, low power dissipation, simple geometry and especially, its wide application range of *Re* number [4,6]. Although it has been popularly used in industrial processes, there is little fundamental research on its distributive mixing mechanisms, not to say theoretical research on dispersive mixing characteristics. The questions that often arise concern the type of MAXBLEND that performs the best in a given situation. Should one opt for a standard type or other ones? Besides this, such questions are often put forward; why does MAXBLEND have excellent dispersive mixing performance? Is the dispersive mixing ability high in all *Re* ranges? Quite clearly, the answer to such questions is non-trivial since the quality of mixing achieved in a vessel depends on various parameters such as geometry of the equipment, fluid rheology and so on. In this study, we answer the above questions through numerical simulation of dispersive mixing in stirring tanks. In order to quantify dispersive mixing, the strength of the elongational flow is calculated and is taken as the local dispersive mixing performance index; the homogenization degree of dispersive mixing is described with an number of passage distribution (NPD) function and the global mixing ability is quantified by the average number of passages. A standard type of MAXBLEND impeller for low and moderate Re is numerically analyzed using the above approach. Finally, the results are compared with those of a double helical ribbon impeller (hereafter, this is called a DHR).

2. Description of numerical methods

2.1. Flow simulation

The mixing vessel used for numerical calculation is cylindrical and has a plated bottom with a diameter of 2 m. Two types of agitators (Fig. 1) will be tested. One is a standard type of MAXBLEND rotating at 15 and 150 rpm, respectively, which gives *Re* numbers of 32.4 and 324, respectively (the material used is a Newtonian fluid with a viscosity of 20 Pa s and a density of 1400 kg/m^3). The other is a DHR impeller rotating at 15 rpm giving a *Re* number of

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1385-8947/01/$ – see front matter
PII: S1385-8947(01)00135-8

Nomenclature

$\boldsymbol{D}$	deformation rate tensor
f_{NPD}	number passage distribution function
g_k	fraction of material volume in batch or continuous system that has experienced k passages
k	number of passages
l	the radii of particles making up cluster
N_i	number of fluid particles
$N_i(r,s,t)$	elements shade function
r, s, t	local coordinates in FEM calculation space
v_i	node velocity
v_{p}	velocity of a particle
x	location of a particle

Greek letters

$\boldsymbol{\Omega}$	vorticity tensor
$\dot{\gamma}$	shear strain
η_{e}	elongational viscosity
η_{s}	shear viscosity
λ	local dispersive mixing efficiency
$\tau_{\mathrm{elongation}}$	elongational stress
τ_{shear}	shear stress

44.9. The detailed size of the stirring tank is also shown in Fig. 1. The simulation of the flow field in the tank is governed by momentum balance and continuity equations, the flow regime is laminar, and the flow field is solved using the three-dimensional FEM software SUNDYFLOW developed by Plamedia Research corp. in Japan. To simplify calculation and to avoid re-meshing, a steady flow field on the rotating frame fixed on the impellers is adopted. In such a frame of reference, the boundary conditions are as follows:

- A no-slip condition on the non-moving impeller;
- The rotational speed on the tank walls.

Besides these, a flat free surface is assumed. In the case of MAXBLEND, 10,800 elements with 12,000 nodes were placed in the analysis regions; in the case of DHR, 10,800 elements and 11,970 nodes were used.

2.2. Computation of particle trajectories

To quantify the dispersive mixing performance, local mixing efficiency is calculated based on a known velocity field, and NPD is computed by tracking large numbers of particles initially distributed evenly on the *Y–Z* plane, from start to end and recording the mixing histories. The particle moves based on the following equation:

$$x(t) = x(0) + \int_0^t v_{\mathrm{p}}(x(t'))\,\mathrm{d}t' \tag{1}$$

$$v_{\mathrm{p}} = \sum_0^M v_i \times N_i(r, s, t) \tag{2}$$

where $x(0)$ is the initial location of a particle and $v_{\mathrm{p}}(x(t'))$ the velocity of the particle located at $x(t')$; v_i the node velocity, and $N_i(r, s, t)$ the element shade function, and M is the nodes number per element.

2.3. Quantitative estimation of dispersive mixing characteristics

2.3.1. Local dispersive mixing efficiency

Traditionally, mixing is studied by analyzing the flow field. More recently, it has been recognized that the velocity field obtained from numerical solutions provides very limited information about the details of a mixing process. Flow properties, such as turbulent energy, disspation rate and velocity vectors, provide only general information on the actual fluid mixing. In order to learn dispersive mixing well, a quantitative description of the dispersive mixing mechanisms is necessary. The deformation and breakup of droplets of the minor component in a two-phase system

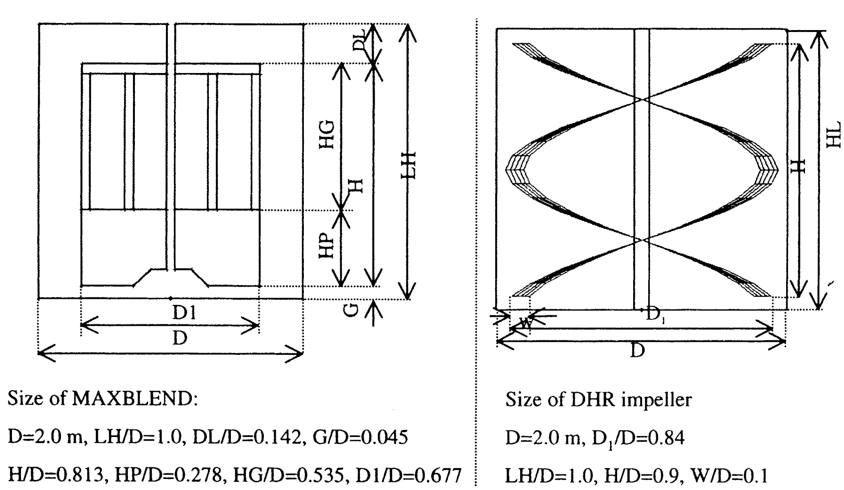

Fig. 1. Geometry of standard type of MAXBLEND and DHR impellers and their sizes for simulation.

have been extensively studied [1,2,7,8] in the past. Quantitative studies of droplet breakup in simple shear and pure elongational flows have shown that elongational flow is more effective, especially in the case of high viscosity ratio and low interfacial tension systems. Manas-Zloczower and Feke have obtained similar conclusions for the dispersion of solid agglomerates into a liquid matrix [5]. One of the key factors in deforming or breaking up a droplet or agglomerates is the capillary number, defined as Ca, which represents the ratio of the shear stress applied to the interfacial tension. If the dispersed phase is solid agglomerates, for simple shear flow, the maximum hydrodynamic force acting on a dumb-bell shaded cluster is given by [9]

$$\tau_{\mathrm{shear}} = 3\pi\eta_{\mathrm{s}}\dot{\gamma}l^2 \quad (3)$$

Whereas for pure elongational flow

$$\tau_{\mathrm{elongation}} = 6\pi\eta_{\mathrm{e}}\dot{\gamma}l^2 \quad (4)$$

where η_{s} is the shear viscosity of the matrix fluid, η_{e} the elongational viscosity, and l the radii of particles making up the cluster. Considering that the elongational viscosity is usually three times the shear viscosity, the equations above indicate that elongational flow can generate substantially higher stresses than shear flow. Therefore, in this study, we can use the relative strength of elongational flow components in the flow field to assess the local mixing efficiency, which is defined as [3]

$$\lambda = \frac{|\boldsymbol{D}|}{|\boldsymbol{D}| + |\boldsymbol{\Omega}|} \quad (5)$$

where $\boldsymbol{D}$ is the deformation rate tensor and $\boldsymbol{\Omega}$ the vorticity tensor. Because the vorticity tensor cannot generate shear stress, therefore, the parameter assumes values between 0 for pure rotation (the worst dispersive mixing performance) and 1 for pure elongation (the highest dispersive mixing efficiency); for simple shear flow, the value of λ equals 0.5.

3. Number of passages distribution function (NPD function)

3.1. Definition of NPD

Consider a batch or continuous mixing system, $g_k(t)$, the fraction of volume in the system that has experienced k passages in a volume region of interest (such as that the stress in the site is larger than one designated value or something else) in time t, can be defined as [10]

$$g_k(t) = \frac{\lambda^k}{k!}\mathrm{e}^{-\varepsilon} \quad (6)$$

where ε is the dimensionless time, $\varepsilon = t/\theta$, where θ is the mean passage time between successive passages. In the same paper Tadmor also derived some NPD functions in some ideal model systems (for details please refer the paper). With the NPD function and mean passages, we can quantitatively know the mixing performance of the mixing system.

3.2. Application of NPD function in dispersive mixing

There are several practical problems in using the NPD function proposed by Z. Tadmor [10] to quantify dispersive mixing in a mixer. The first is that a pre-assumption of a real mixer using an appropriate ideal model such as a well-stirred vessel with recirculation (for a closed mixing system) or a plug flow with recirculation (for an open mixing system) is necessary. The second is that the "passages" originally means passing through high stress regions; however, which values of stress in a mixer are considered high? It is impossible to give an absolute threshold value of shear stress for all mixers. For these reasons, although the merits of using NPD to quantify dispersive mixing was discussed by Tadmor about 10 years ago, there have been few application examples. In this study, we try to numerically calculate the NPD function by the following methods.

1. Instead of pre-assumption models, we directly calculate the NPD function using particle simulation method; through tracing the dispersive mixing histories of these massless particles, the NPD can be obtained statistically. The NPD function can be calculated as follows:

$$f_{\mathrm{NPD}} = \frac{N_i(i \to i+1)}{\sum_{i=0}^{i=i_{\max}} N_i} \quad (7)$$

   where N_i is the number of fluid elements which pass through the high dispersive regions for i times. From the NPD function, we can determine the homogeneity of the minor phase in the matrix.
2. Instead of using the shear stress value as the judgement standard of "passage", based on previous experimental and theoretical results as described above, we use the relative strength of elongational flow in the flow field as the judgement standard of dispersive mixing performance. According to the experimental results of Bentley and Leal [2], when the viscosity ratio of the dispersed phase to matrix is between 0.1 and 10, the integrated flow of simple shear and somewhat strong elongational flow ($\phi = 0.6$) is able to break a drop up at low Capillary number. Therefore, if a fluid element passes through the region where the local dispersive mixing efficiency surpasses 0.6, one passage is counted.

4. Results and discussion

4.1. Local dispersive mixing efficiency

Fig. 2 shows the local dispersive mixing efficiency distribution and the velocity vector in the X–Z cross-section where MAXBLEND is included. We cannot obtain any information on dispersive mixing performance from the velocity vector. The local dispersive mixing efficiency distribution in this figure indicates that near the regions of the grid part of a MAXBLEND higher local mixing efficiency

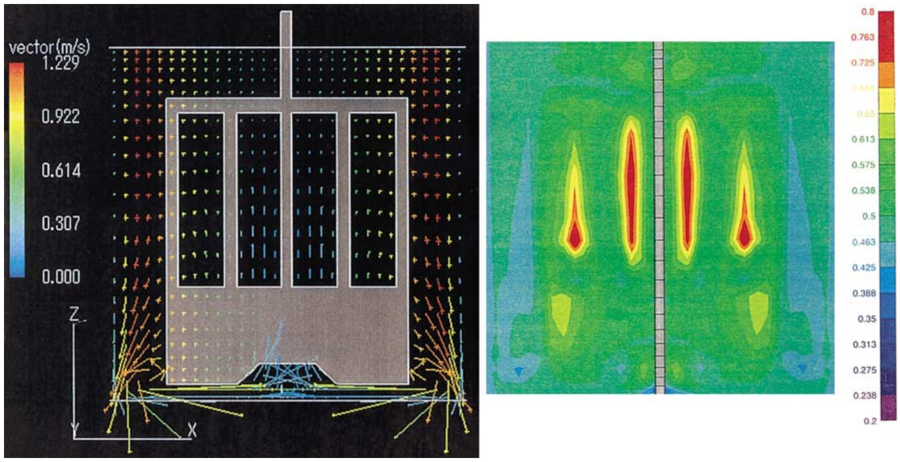

Fig. 2. Local dispersive mixing efficiency and velocity vector in the *X–Z* plane of a stirring tank with a standard type of MAXBLEND ($Re = 32.4$).

(high to 1, pure elongation flow) is produced. Droplets or solid agglomerates are broken up efficiently when they flow through those regions. In contrast to this, in the outer paddle regions, low local mixing dispersive efficiency exists; however, the regions having low dispersive mixing efficiency are small. The results also point out that dead-spots in dispersive mixing exist in the central bottom of the impeller. Fig. 3 shows the local mixing efficiency distribution in the cross-section cut perpendicular to the above plane but with the same axis, which also indicates that the dispersive mixing efficiency near the grid regions, especially the inner grid regions is high and low mixing efficiency regions occur in the outer paddle regions. In short, in most of the regions of a stirring tank, MAXBLEND gives satisfactory local dispersive mixing efficiency. Compared with this, the local mixing efficiency in a stirred tank with a DHR impeller as shown in Fig. 4 is disappointing, which illustrates the local mixing efficiency distribution in the *X–Z* plane. The highest value of local mixing efficiency is <0.63, which is a very low value compared with that in a stirring tank with a MAXBLEND impeller. Only small regions under the ribbon impeller and those close to the tank bottom have high dispersive mixing efficiency. In contrast to this, the rotation flow component dominates most of the regions. As mentioned above, rotation flow contributes nothing to the dispersive process; therefore, the dispersive ability of the DHR impeller is not high.

5. NPD function

As mentioned in the first section, only by local mixing efficiency we cannot reach any decision on whether or not an impeller's dispersive mixing performance is good, because the final dispersive mixing performance of an impeller is due to homogenization of the size of dispersed phase. In this study, the homogenization degree is described by the NPD function as shown in Fig. 5, which is calculated by tracking 500 particles and counting each passage while the particle flows through high dispersive mixing efficiency regions. (During numerical simulation, several tests with different

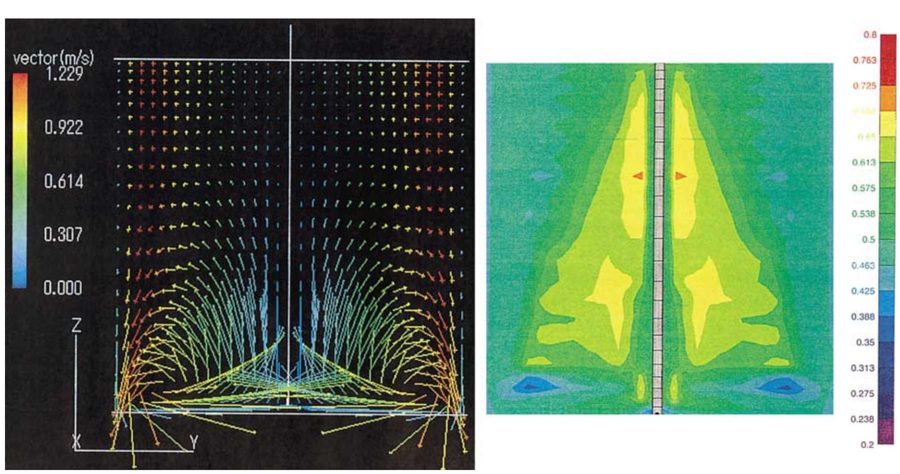

Fig. 3. Local dispersive mixing efficiency and velocity vector in the *Y–Z* plane of a stirring tank with a standard type of MAXBLEND ($Re = 32.4$).

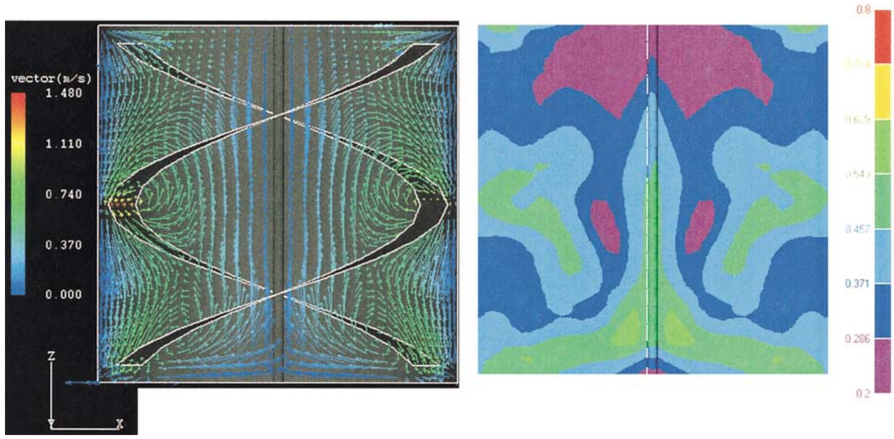

Fig. 4. Local dispersive mixing efficiency and velocity vector in the *X*–*Z* plane of a stirring tank with a DHR impeller ($Re = 44.9$).

numbers of particles were carried out to obtain a convergent curve. Finally, the minimum number of particles of 500 for the converged curve is used). The tracking time for impellers with 15 rpm is 70 s, and for impellers with 150 rpm the time is 7 s. From this figure, we learn that the NPD function of a DHR impeller, although with about 32% of the particles not flowing through the high dispersive region for even one time, has a narrow breadth. The narrow distribution means two things. Firstly, the particles tracked were imposed near a similar dispersive mixing history. Secondly, the total circulation of particles in a stirring tank with DHR impellers is good, which can also be found by tracking the trajectory of a passive particle initially located at any place in the tank as shown in Fig. 6. In this figure, a total time 150 s is used to track the one particle; from this figure we may see that the particle goes around most of the regions in the tank. This means that if high dispersive regions existed in the tank, the chance for each particle passing through the regions would be identical. Therefore, we can conclude that the size of the dispersed phase will have a sharp distribution if coalescence of the dispersed phase is not considered. However, the average size may be large as the average number of passages is only three within 70 s.

In the third section, through calculation of the local dispersive mixing efficiency we learn that the stirring tank with a standard type MAXBLEND provides a satisfactory local dispersive mixing efficiency. However, despite the fact that very efficient dispersive regions are produced by the special geometry of the grid part, when a low rotation speed of MAXBLEND (15 rpm) is used, the NPD as shown in Fig. 6 is not as satisfactory as we expected. Although the maximum number of passages is as high as 28, about 40%

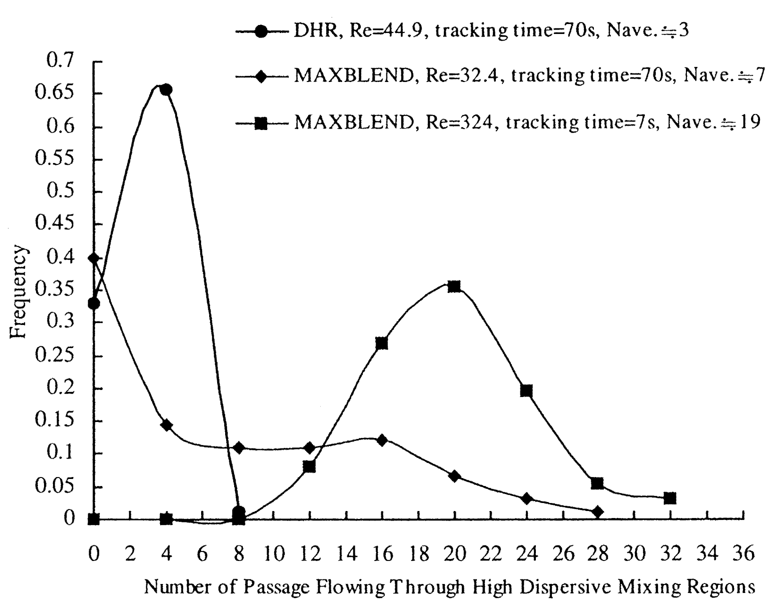

Fig. 5. NPD functions for different impellers and/or different Reynolds number.

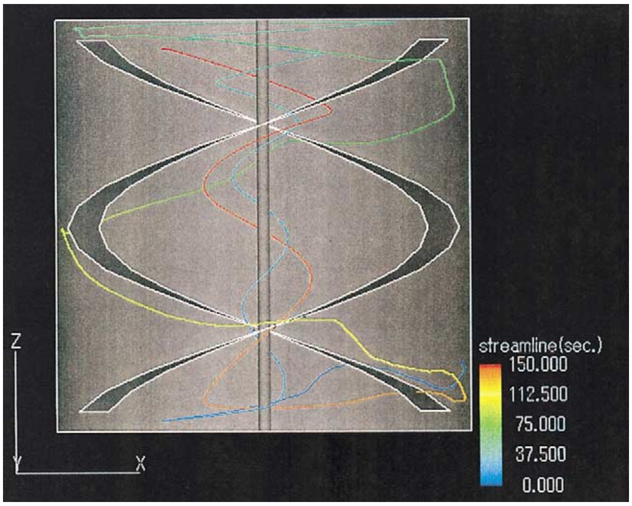

Fig. 6. One particle trajectory tracked for 150 s in a stirring tank with the DHR impeller.

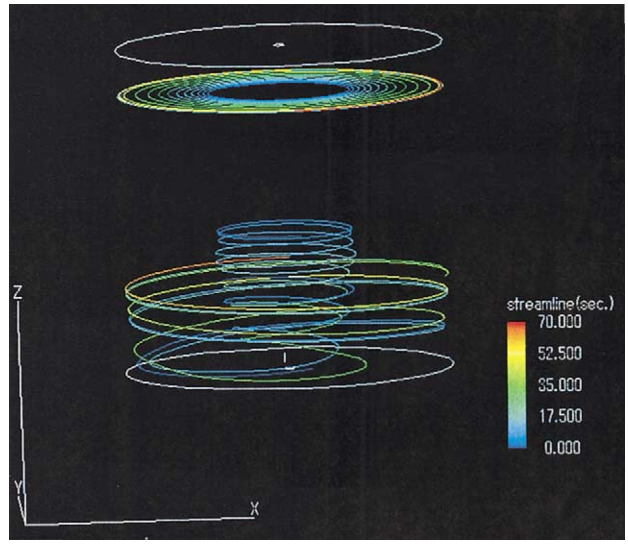

Fig. 7. Three particle trajectories tracked for 70 s in a stirring tank with the standard type of MAXBLEND impeller ($Re = 32.4$).

particles does not go through high dispersive regions even once and the breadth of the NPD is very wide. Certainly, under such an operating condition, the size distribution of the dispersed phase will be disappointing. The reason can also be found from the particle pathline simulation as shown in Fig. 7. Here, three particles initially located at different places were tracked for 70 s, from the trajectories we can learn that because of a weak axial flow, the particle initially located at the upper part of the tank rotates around the central axis with very weak up or down movement. Although the axial flow near the bottom of the tank is a little stronger than that in the upper, it is not strong enough to move all the fluids in the bottom of the tank to the grid part, not to say to the upper part of the tank. The weak axial flow can also be learned from the velocity vector as shown in Figs. 2 and 3. Therefore, a good total circulation in all the regions of the tank cannot be achieved, which results in an inferior NPD. In contrast to this, with the same impeller, but with the rotation speed increased to 150 rpm, although the local dispersive mixing efficiency pattern shows no great change compared with that obtained in the low rotation speed, a satisfactory NPD function can be obtained as shown in Fig. 5. Despite the fact that the tracking time is only 7 s, the average number of passages is as high as 19 and the breadth of the NPD function is relatively narrow. The result can also be explained from the particle trajectory as shown in Fig. 8. In this figure, a particle initially located at the bottom of the tank was tracked for 15 s. During this period, the particle went up and down three times but by different path lines. When it went up, it repeatedly went through the grid part where it will be stretched and broken up if it is a droplet or

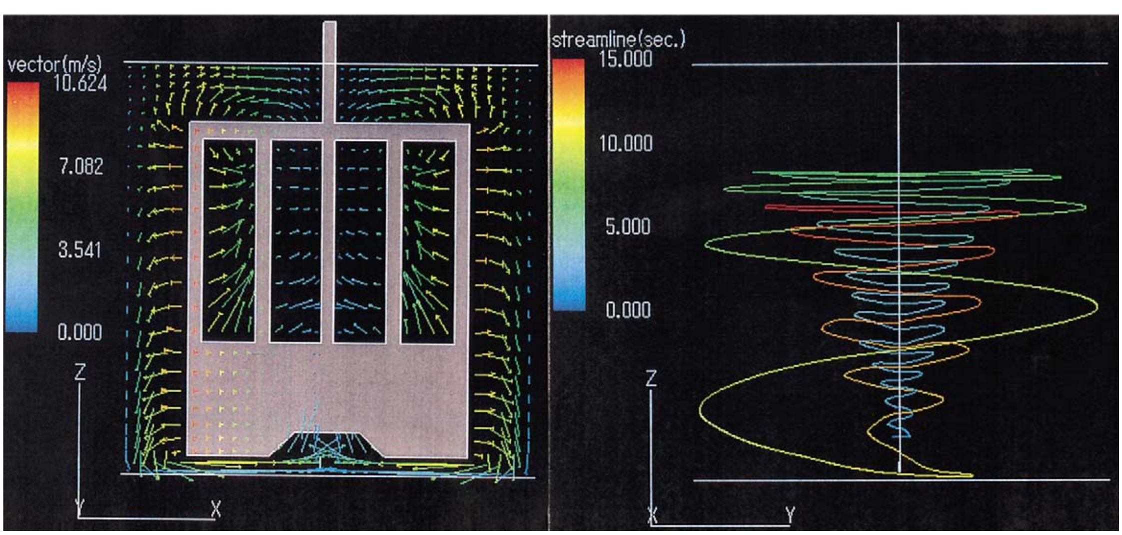

Fig. 8. Velocity vector and one particle trajectory tracked for 15 s in a stirring tank with the standard type of MAXBLEND impeller ($Re = 324$).

a solid agglomerate. From only this one particle trajectory, we can learn that under such an operating condition the total circulation in the tank is good; this is the necessary condition for a good NPD. A good total circulation plus a promising local dispersive mixing efficiency distribution will give an ideal NPD.

6. Evaluation of dispersive mixing performance for MAXBLEND and DHR

By integrating the local dispersive mixing efficiency distribution and the NPD function we can create a rational estimation of the dispersive mixing performance for the analyzed impellers. Based on the above numerical results for the two factors, we can conclude that the DHR impeller is not an ideal dispersive mixer because it cannot produce strong elongational flow in the stirring tank despite the fact that it can produce a narrow size distribution of dispersed phase because of the good total circulation of fluid in the tank. The standard type MAXBLEND provides an ideal local dispersive mixing efficiency distribution because of its special design of the grid part in the impeller, which can induce strong elongational flow, and in some places, nearly pure elongational flow. However, it seems that only under a moderate or more higher Reynolds number can the standard type MAXBLEND produce a good total circulation of fluid, and therefore, an ideal size distribution of the dispersed phase and an fine droplets or agglomerates. At a low *Re* number, although local dispersive mixing efficiency distribution has a pattern similar to that at a moderate *Re*, because of the weak axial flow, the total circulation of fluid is not promising, and the high dispersive regions cannot be efficiently utilized. Therefore, the standard type MAXBLEND with a low *Re* number is not appropriate for the dispersive mixing process.

7. Conclusions

Dispersive mixing performance in a stirring tank with a standard type MAXBLEND and a DHR impellers has been numerically analyzed using local dispersive mixing efficiency and the NPD function. The flow field was solved using FEM methods, and strength of the elongational flow component in the flow field is used as an index to judge the local dispersive mixing performance. A large number of passive particles were tracked to record whether and how many times they flowed through the high dispersive regions and to obtain the NPD function. The numerial results are as follows:

1. Although the total circulation of fluid in a tank with a DHR impeller is good, because the elongational flow induced by DHR is weak, therefore, the DHR is not a promising dispersive mixer.
2. A standard type MAXBLEND has the ability to produce a strong elongational flow, especially in the grid part. When operated under a moderate Reynolds number, the standard type MAXBLEND is a very effective dispersive mixing impeller. However, when the Reynolds is low, the local high dispersive performance cannot be utilized effectively because of the unsatisfied total circulation of fluid resulting from the weak axial flow.
3. The conclusions are based only on a numerical simulation; the validity of the conclusions must be justified by experimental study. This is our future work.

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